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Backreaction in Closed String Tachyon Condensation

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ABSTRACT: We consider backreaction due to production of massless strings in the background of a condensing closed string tachyon. Working in the region of weak tachyon, we find the modified equations of motion for massless strings with conformal perturbation theory. We solve for the positive and negative frequency modes and estimate the backreaction on the background dilaton. In large (supercritical) dimensions, we find that the backreaction can be significant in a large region of spacetime. We work with the bosonic string, but we expect these results to carry over into the heterotic case.

KEYWORDS: Tachyon Condensation,Bosonic Strings.

Contents

1. Introduction

Tachyon condensation in various string constructions has become increasingly important as a set of clean examples of time-dependent systems in string theory and as a dynamical connection between varying states of string theory. The most famous example is condensation of the open string tachyon on either an unstable D-brane or a D-brane/anti-D-brane pair; Sen's conjecture, which has by now considerable supporting evidence, states that tachyon condensation annihilates the branes and leaves a purely closed string state behind (see [1–4] for reviews). In fact, recent work has found an exact dynamical solution connecting the unstable brane system to the closed string vacuum in string field theory [5].

Closed string tachyons have also generated a great deal of interest, beginning with tachyons localized at special points in space [6–18]. For example, tachyons can develop at orbifold fixed points with conical singularities; tachyon condensation appears to reduce the rank of the orbifold group and eventually resolve the singularity.

Of course, bosonic string theory (and various nonsupersymmetric heterotic theories) has a nonlocalized "bulk" closed string tachyon. Bulk closed string tachyon condensation seems to realize the conjecture of [19–21] that the closed string tachyon "vacuum" is a bubble of nothing along the lines of [22]. Bulk tachyon condensation has been studied in [23–35];

roughly speaking, the tachyon generates a mass for some of the worldsheet fields, effectively turning off propagation and oscillation of strings in the corresponding dimensions of spacetime. Therefore, as the tachyon condenses, spacetime loses dimensionality; central charge is unchanged during tachyon condensation because the dilaton background changes at the same time [25]. Perhaps the natural tachyon profile to consider is exponential growth in a timelike direction; these tachyon profiles have α' corrections that are suppressed in supercritical string theories with large dimensionality D [25]. However, the cleanest results have been achieved in backgrounds in which the tachyon grows exponentially along a lightlike direction; this tachyon profile is an exact solution of perturbative string theory at all orders in α' [28–30]. (Of course, there are still corrections at nonzero string coupling.)

Compared to these impressive results in worldsheet physics, our understanding of closed string tachyon condensation in spacetime is somewhat behind. Some progress has been made in developing the effective field theory of massless closed strings and tachyons, including some solutions of that theory [36–38], though a fully consistent action has only been described recently [39]. Nonetheless, it has been proposed that tachyon condensation could provide a stringy resolution of cosmological singularities [16, 40]. This proposal was taken a step farther in [41], which assumed the Big Bang could be replaced by emergence from a tachyon condensate and asked what quantum mechanical perturbations are generated by the time dependence of the tachyon; these fluctuations could potentially serve as the initial values for inflationary perturbations. That paper used a somewhat heuristic worldsheet Hamiltonian approach to calculate particle production and found that the late-time fluctuations are a thermal state at the energy scale set by the tachyon gradient.[1]

In this paper, we reconsider particle production in a background of tachyon condensation, working in the usual picture that the tachyon grows toward the future. Our goal is to determine if quantum fluctuation of massless string states, in particular the dilaton, backreact significantly on known classical tachyon backgrounds. We proceed by using conformal perturbation theory to determine how massless strings propagate in a weak tachyon background. We then solve the modified equation of motion and calculate the quantum source for the dilaton (analogous to finding the quantum mechanical stress-energy tensor in a cosmological background). We find that backreaction can be large even when most g_s corrections are small. Although we focus on backreaction in this paper, the reader should note that the amplitude calculation we have done (and similar calculations) are useful for confirming the effective action of [39].

The plan of this paper is as follows. In section 2, we review the tachyonic backgrounds of bosonic string theory that we consider. We also show directly that the lightlike tachyon background is a conformal field theory on the worldsheet.[2] Then, in section 3, we calculate the scattering amplitude of a tachyon and two massless string states. (The appendix contains a review of the BRST quantization of the string in the linear dilaton background, which is useful for the calculation of this amplitude.) In conformal perturbation theory, this amplitude

[1] Cosmological questions relevant to open-string tachyon condensation have recently been addressed in [42].

[2] We thank S. Hellerman for his patient explanation of this calculation.

gives the modified propagator of the massless states in a weak tachyon background, which directly tells us the modified equation of motion for perturbations of the massless fields. Finally, in section 4, we calculate the quantum mechanical source for the dilaton, which we show to be large in many circumstances.

2. Review of Bulk Tachyon Condensation

In this section, we review known results about closed string tachyon condensation from the worldsheet perspective. Rather than consider tachyons localized at some point in the spacetime (such as a shrinking circle or a nonsupersymmetric orbifold fixed point), we will restrict our attention to the bulk closed string tachyons of the bosonic theory. We will mostly discuss tachyons with a lightlike gradient as discussed in [28–30], since they are uncorrected in α', but we will also show how our discussion carries over for tachyons with a timelike gradient.

We begin by reminding the reader of allowed tachyon backgrounds in supercritical string theory, largely following [25,28]. Then we show explicitly that the conformal anomaly vanishes for the lightlike tachyon gradient. As far as we know, this proof has been known for some time but has not appeared in the literature [43].

A brief note on conventions: we take lightcone coordinates so that the Minkowski metric is

$$ds^2 = -(dX^0)^2 + (dX^1)^2 + d\vec{X}^2 = -2dX^+dX^- + d\vec{X}^2 \ . \tag{2.1}$$

2.1 Tachyon Vertex Operator

In order to control worldsheet corrections to tachyon condensation, we work in a linear dilaton background, typically in large supercritical dimension D. (In fact, we will typically consider the tachyon to be a small perturbation to the linear dilaton.) In the string frame (*i.e.*, the spacetime fields that couple to the string variables), the metric is Minkowski and the dilaton takes the form $\Phi = V_\mu X^\mu$ with $V^2 = (26 - D)/6\alpha'$. The linear dilaton background is a well-known exact solution of tree-level string theory [44]. We choose the dilaton to decrease into the future, $V_0 = -\sqrt{(D-26)/6\alpha'}$. We can also consider a tachyon background in the critical dimension $D = 26$, in which case we take only V_- to be nonzero; typically, though, we will work in supercritical dimension.

As has been observed, for example in [25], the condition for a tachyon vertex operator to have the correct weight in a general linear dilaton background is

$$\partial_\mu\partial^\mu T(X) - 2V^\mu\partial_\mu T(X) + \frac{4}{\alpha'}T(X) = 0 \ . \tag{2.2}$$

Due to the tachyonic mass term, the solution will have some time dependence, which we take to be exponential in either X^0 or X^+.

It is easy to check that

$$T(X) = \frac{\mu_0^2}{2}e^{\beta X^+} \tag{2.3}$$

solves (2.2) for $\beta = 2/V^+\alpha'$. In most string theories, this solution represents a bubble of nothing.[3] Of more interest to us is a tachyon with some spatial dependence,

$$T(X) = \frac{\mu^2}{2\alpha'} e^{\beta X^+} {:}(X^a)^2{:} + \delta T(X) \ . \tag{2.4}$$

Here, $(X^a)^2$ is a sum over N of the transverse spatial directions (rather than all the spatial dimensions). (The reader should note that these are *transverse* dimensions, so we necessarily have $N \leq D-2$.) We include δT in order to solve the equation of motion; then we find

$$\delta T(X) = \frac{N\mu^2}{4} \left(\beta X^+\right) e^{\beta X^+} \tag{2.5}$$

for the specific solution (of course, (2.3) can be added independently), as explained in [29].

The tachyon with quadratic spatial dependence (2.4) can be written as the long-wavelength limit of a plane wave, as in [29]. In particular,

$$T(X) = \mu^2 e^{\beta_k X^+} {:}e^{i\vec{k}\cdot\vec{X}}{:} \ , \quad V^+\beta_k = \frac{2}{\alpha'} - \frac{1}{2}\vec{k}^2 \tag{2.6}$$

is a valid tachyon background. Without loss of generality, we can take $\vec{k}$ to point along a single axis (say the Y direction), in which case (2.4,2.5) (with $N=1$) follow by taking the $k \to 0$ limit of

$$T(X) = \mu_0^2 \left(e^{\beta X^+} - e^{\beta_k X^+} {:}\cos(kY){:} \right) \tag{2.7}$$

with $\mu^2 = \alpha' \mu_0^2 k^2$ fixed. Incidentally, (2.6) shows immediately that, even in the critical dimension, the lightlike tachyon only solves the equations of motion in a linear dilaton background.

The story is very similar with a timelike tachyon gradient [25]. The tachyon profile at fixed momentum is

$$T(X) = \mu^2 e^{\beta_k X^0} {:}e^{i\vec{k}\cdot\vec{X}}{:} \ , \quad \beta_k^2 - 2V_0\beta_k = \frac{4}{\alpha'} - \vec{k}^2 \ . \tag{2.8}$$

In the large D limit, we can drop the first term in the quadratic equation for β_k, finding exactly the same dependence as in (2.6). Again, as long as we work in the large dimension limit, we can take the same $\vec{k} \to 0$ limit to find a background

$$T(X) = \frac{\mu^2}{2\alpha'} e^{\beta X^0} {:}(X^a)^2{:} + \frac{N\mu^2}{4} \left(\beta X^0\right) e^{\beta X^0} \ , \tag{2.9}$$

where $\beta V_0 = -2/\alpha'$. These tachyon backgrounds do have α' corrections, but those corrections are suppressed at large D.

Finally, the reader should note that the tachyon "amplitude" μ^2 can be either positive or negative sign, depending on which way the tachyon rolls off its local maximum. We have chosen to write the amplitude as a squared mass to make dimensional factors work out more easily.

[3] As was explained in [30], this tachyon background beginning in the type 0 string is a transition to the bosonic string theory.

2.2 Conformal Invariance

We now remind the reader that the interacting worldsheet theory in the presence of the tachyon is actually a conformal field theory for the lightlike tachyon gradient. It seems that this proof has been known for some time but has not yet made its way into the literature [43]. For simplicity, we work with the $\vec{k} \to 0$ limit of (2.4,2.5).

To get a string background with a nontrivial tachyon, we can (following the usual procedure) just exponentiate the tachyon vertex operator in the linear dilaton CFT. The Polyakov action then becomes

$$S = -\frac{1}{4\pi\alpha'}\int d^2\sigma\sqrt{-\gamma}\left[\partial_\alpha X^\mu \partial^\alpha X_\mu + \frac{\mu^2}{2}e^{\beta X^+}\left((X^a)^2 + \frac{N\beta\alpha'}{2}X^+\right)\right] + \frac{\{i\}}{4\pi}\left(\int d^2\sigma\sqrt{-\gamma}RV_\mu X^\mu + 2\int_{\partial M} d\sigma K V_\mu X^\mu\right) . \tag{2.10}$$

We have included the contribution from the linear dilaton just to remind the reader that it is there; the worldsheet boundary includes "boundaries at infinity." At this stage, it doesn't matter whether we are using a Euclidean or Lorentzian worldsheet; the factor of i in curly braces is appropriate for the Lorentzian case.

Let us make more explicit the correct renormalized (UV finite) form of the action. Beyond the normal divergences of the free theory, the tachyonic theory only gets UV divergences from loops with a single propagator (from [29], we know that there are no diagrams beyond one loop), as illustrated in figure 1. With a point-splitting regulator, the divergence comes from the coincidence of the two ends of the propagator. In this limit, then, the divergence is the same as in $\mathbb{R}^{1,1}$, namely

$$-\frac{N\alpha'\mu^2}{4}e^{\beta X^+}\ln\left(\Delta s^2\right) , \tag{2.11}$$

where Δs^2 is the proper distance between the two ends of the propagator (we have chosen the coordinate invariant form for obvious reasons).

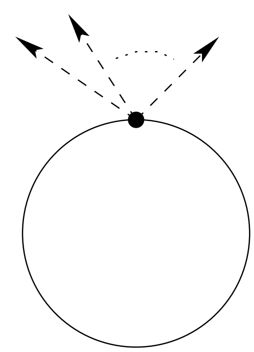

Figure 1: The only diagrams contributing divergences are those with a single vertex with an arbitrary number of X^+ lines leaving and one X propagator in a loop.

To write the renormalized action, then, we only need to replace $(X^a)^2$ with the conformally normal ordered version $:(X^a)^2:$, just as we would guess from the operator treatment. (Note that the free action should technically replace $(\partial X)^2$ similarly with $:(\partial X)^2:$). This renormalized action is the proper starting point for the Hamiltonian treatment described in [41].

By taking the variation of the action with respect to the metric, we find the stress tensor

$$T_{\alpha\beta} = -\frac{1}{\alpha'}\left(:\partial_\alpha X^\mu \partial_\beta X_\mu: -\frac{1}{2}\gamma_{\alpha\beta}:\partial_\gamma X^\mu \partial^\gamma X_\mu:\right) + V_\mu \nabla_\alpha \nabla_\beta X^\mu - \gamma_{\alpha\beta}V_\mu \nabla^2 X^\mu + \frac{\mu^2}{4\alpha'}\gamma_{\alpha\beta}\, e^{\beta X^+}\left(:(X^a)^2: + \frac{N\beta\alpha'}{2}X^+\right) + \frac{N\mu^2}{4}e^{\beta X^+}\frac{\Delta\sigma_\alpha \Delta\sigma_\beta}{\Delta s^2} . \tag{2.12}$$

The first line of (2.12) is the usual contribution from the kinetic term and the linear dilaton, while the second line gives the classical and quantum contributions from the tachyon (potential) term. Note that the last, quantum mechanical, term arises from the metric dependence in $:X^2:$. When properly averaged over possible directions, $\Delta\sigma_\alpha\Delta\sigma_\beta \to (1/2)\gamma_{\alpha\beta}\Delta s^2$. The reader should also be aware that a similar quantum mechanical term should arise from the normal ordering of the kinetic terms, but it cancels out.

Conformal invariance is simple enough to demonstrate. The trace of the stress tensor is just

$$T^\alpha_\alpha = -V_-\nabla^2 X^- + \frac{\mu^2}{2\alpha'}e^{\beta X^+}\left(:(X^a)^2: + \frac{N\beta\alpha'}{2}X^+ + \frac{N}{2}\right) . \tag{2.13}$$

We can simplify this expression using the equation of motion for X^-, which is

$$2\nabla^2 X^- = -\frac{\beta\mu^2}{2}e^{\beta X^+}\left(:(X^a)^2: + \frac{N\beta\alpha'}{2}X^+ + \frac{N\alpha'}{2}\right) . \tag{2.14}$$

Along with $V_- = -2/\beta\alpha'$, this yields $T^\alpha_\alpha = 0$.

In terms of worldsheet lightcone coordinates on a flat worldsheet, we find the usual stress tensor

$$\begin{aligned} T_{++} &= -\frac{1}{\alpha'}:\partial_+ X^\mu \partial_+ X_\mu: + V_-\partial_+^2 X^- \\ T_{--} &= -\frac{1}{\alpha'}:\partial_- X^\mu \partial_- X_\mu: + V_-\partial_-^2 X^- . \end{aligned} \tag{2.15}$$

As usual, the central charge from the total dimensionality, linear dilaton, and conformal ghosts cancel. The tachyon does not contribute to the central charge at all.

3. Propagation in the Weak Tachyon Region

In this section, we compute the string perturbation theory amplitude for a single tachyon and two massless strings in a linear dilaton background. This amplitude provides the first order correction to the massless string action in the tachyon background. We can then see how the massless mode propagation differs in the presence of a weak tachyon. In this section, we work on a Euclidean worldsheet with complex coordinates z, and we work at tree level in string perturbation theory.

3.1 Tachyon Modification to Action

The propagation of massless string modes will be modified by scattering off the tachyon background; the one-tachyon/two-massless-string amplitude gives the shift in the two-point massless correlation function and therefore the action for the massless strings. Since we want to think of massless string scattering from the tachyon background, at lowest order we should calculate the string diagram with three vertex operators. (Higher order calculations in the tachyon amplitude correspond to adding more tachyon vertex operators.)

The two massless string vertex operators are given by

$$e^{1,2}_{\mu\nu} :\partial X^\mu \bar{\partial} X^\nu e^{ik_{1,2}\cdot X}: (z_{1,2}, \bar{z}_{1,2}) \ , \tag{3.1}$$

in which the polarization tensors and momenta obey the gauge conditions given in (A.9). In particular, the mass-shell and gauge conditions are

$$k_{1,2} \cdot (k_{1,2} + 2iV) = 0 \ , \quad e^{1,2}_{\mu\nu}(k_{1,2} + 2iV)^\nu = e^{1,2}_{\nu\mu}(k_{1,2} + 2iV)^\nu = 0 \ . \tag{3.2}$$

For ease of calculation, we will use the plane-wave tachyon vertex operator (2.6), and we will combine the X^+ and X^i exponentials into a single exponential with relativistic momentum k^3_μ. We will not concern ourselves with the overall scaling of the vertex operators, instead normalizing the final shift of the action to get the correct dimensionality.

Since there are only three vertex operators, we work with the fixed-position form of the amplitude (up to normalization)

$$\mathcal{A} = e^1_{\mu\nu} e^2_{\lambda\rho} \left\langle :\partial X^\mu \bar{\partial} X^\nu e^{ik_1\cdot X}: (z_1, \bar{z}_1) :\partial X^\lambda \bar{\partial} X^\rho e^{ik_2\cdot X}: (z_2, \bar{z}_2) :e^{ik_3\cdot X}: (z_3, \bar{z}_3) \right\rangle \times (\text{ghosts}) \ . \tag{3.3}$$

The vertex operator positions $z_i \in \mathbb{C}$ are arbitrary fixed positions on the worldsheet. The path integral over the X^μ zero modes gives a momentum preserving delta-function; however, in the linear dilaton background, the sum of momenta also includes the dilaton gradient V^μ (see, for example, exercises in [44]). Therefore, we have

$$\mathcal{A} \propto (2\pi)^D \delta^D \left(\sum_i k_i + 2iV \right) \ . \tag{3.4}$$

The ghost contribution is unchanged from that in the Minkowski background of critical string theory. The expectation value can be simplified through the use of (3.2) to read

$$\begin{aligned} \mathcal{A} &= (2\pi)^D \delta^D \left(\sum_i k_i + 2iV \right) e^1_{\mu\nu} e^2_{\lambda\rho} \, |z_{12}|^{\alpha' k_1\cdot k_2} \, |z_{13}|^{\alpha' k_1\cdot k_3} \, |z_{23}|^{\alpha' k_2\cdot k_3} \\ &\quad \times \left(\eta^{\mu\lambda} - \frac{\alpha'}{8} k^\mu_{23} k^\lambda_{13} \right) \left(\eta^{\nu\rho} - \frac{\alpha'}{8} k^\nu_{23} k^\rho_{13} \right) \ , \end{aligned} \tag{3.5}$$

where $z_{ij} = z_i - z_j$ and similarly for k_{ij}.

We get the physically meaningful amplitude (again, up to dimensionful normalizations) by using a coordinate transformation to choose values of the vertex operators. A particularly convenient choice is $z_1 = 0$, $z_2 = 1$, $z_3 \to \infty$; this limit is well defined due to momentum conservation and the appropriate mass-shell conditions. We end up with

$$\mathcal{A} = (2\pi)^D \delta^D \left(\sum_i k_i + 2iV \right) e^1_{\mu\nu} e^2_{\lambda\rho} \left(\eta^{\mu\lambda} - \frac{\alpha'}{8} k^\mu_{23} k^\lambda_{13} \right) \left(\eta^{\nu\rho} - \frac{\alpha'}{8} k^\nu_{23} k^\rho_{13} \right) \ . \tag{3.6}$$

In the effective theory of the massless strings, this amplitude just corresponds to a shift in the quadratic part of the action. We will specialize to the graviton and dilaton. In that case, the polarization tensor becomes

$$e^{1,2}_{\mu\nu} = h_{\mu\nu}(k_{1,2}) + \gamma\phi(k_{1,2})\eta_{\mu\nu} \ , \tag{3.7}$$

where γ is a numerical coefficient. We can now Fourier transform back to spacetime to get the change in the quadratic action in the presence of a tachyon background. We keep all terms up to second order in derivatives and find

$$\begin{aligned}\Delta S = \mu^2 \int d^D x\, e^{-2V\cdot X} e^{ik\cdot X} \Big[& 4\left(h_{\mu\nu} + \gamma\phi\eta_{\mu\nu}\right)\left(h^{\mu\nu} + \gamma\phi\eta^{\mu\nu}\right) + \alpha'(\partial+ik)^\nu h_{\mu\lambda}(\partial+ik)^\mu h_\nu{}^\lambda \\
&+2\gamma\alpha'(\partial+ik)^\mu h_{\mu\nu}(\partial+ik)^\nu\phi + \gamma^2\alpha'(\partial+ik)_\mu\phi(\partial+ik)^\mu\phi \\
&-\frac{1}{4}\alpha'^2 k^\lambda(\partial+ik)^\nu h_{\mu\lambda}k^\rho(\partial+ik)^\mu h_{\nu\rho} - \frac{1}{2}\gamma\alpha'^2 k\cdot(\partial+ik)h_{\mu\nu}k^\mu(\partial+ik)^\nu\phi \\
&-\frac{1}{4}\gamma^2\alpha'^2 k\cdot(\partial+ik)\phi k\cdot(\partial+ik)\phi - \frac{1}{8}\alpha'^2 h_{\mu\nu}(\partial+ik)^\mu(\partial+ik)^\nu h_{\lambda\rho}k^\lambda k^\rho \\
&-\frac{1}{8}\gamma\alpha'^2 k^2 h_{\mu\nu}(\partial+ik)^\mu(\partial+ik)^\nu\phi - \frac{1}{8}\gamma\alpha'^2\phi(\partial+ik)^2 h_{\mu\nu}k^\mu k^\nu \\
&-\frac{1}{8}\gamma\alpha'^2 k^2\phi(\partial+ik)^2\phi - \frac{i}{4}\alpha'^2 h_{\mu\nu}k^\mu(\partial+ik)^\nu h_{\lambda\rho}k^\lambda k^\rho - \frac{i}{4}\gamma\alpha'^2\phi k\cdot(\partial+ik)h_{\mu\nu}k^\mu k^\nu \\
&-\frac{i}{4}\gamma\alpha'^2 k^2 k^\mu h_{\mu\nu}(\partial+ik)^\nu\phi - \frac{i}{4}\gamma^2\alpha'^2 k^2\phi k\cdot(\partial+ik)\phi\Big] \ .\end{aligned} \tag{3.8}$$

We have restored prefactors, choosing them to get a dimensionless action; $h_{\mu\nu}$ and ϕ are canonically normalized in D dimensions, and μ is the mass scale of the tachyon background. We have also dropped the subscript "3" on the tachyon momentum for notational convenience. Notice that the action (3.8) is complex; this is not surprising because we are so far working with a complex plane wave tachyon background. Once we convert to a real tachyon background, the action will be real.

3.2 Modified Equations of Motion

Since we are interested in the propagation of particles through the tachyon background, we now turn to the equations of motion, which has an effect even at the linear level. Since the key physics is the same for both the graviton and the dilaton, we will focus henceforth on the (slightly simpler) dilaton equation of motion. In addition, we will set graviton fluctuations to zero from this point; while dilaton fluctuations necessarily source graviton fluctuations, this mixing is not new to tachyonic backgrounds. Part of the mixing of the dilaton and graviton is due to working in string frame, and part arises already in the pure linear dilaton background. In order to focus on the new physics, then, we just set the graviton fluctuations to zero.

The new contribution to the dilaton equation of motion from (3.8) is then

$$\frac{\delta(\Delta S)}{\delta\phi} = \gamma^2\mu^2 e^{ik\cdot X}e^{-2V\cdot X}\left[8D\phi - \alpha'(\partial - 2V)\cdot(\partial+ik)\phi\right.$$

$$+\frac{1}{2}\alpha'^2 k\cdot(\partial-2V)k\cdot(\partial+ik)\phi-\frac{1}{8}\alpha'^2k^2(\partial+ik)^2\phi$$
$$\left.-\frac{1}{8}\alpha'^2k^2(\partial-2V)^2\phi-\frac{i}{4}\alpha'^2k^2k\cdot(\partial+ik)\phi+\frac{i}{4}\alpha'^2k^2k\cdot(\partial-2V)\phi\right] \ . \quad (3.9)$$

We should now switch over to a real background for the tachyon. Since we are most interested in the time dependence of the system, we will take the long wavelength limit of (2.4,2.5) and (2.9) for the lightlike and timelike tachyon gradients respectively. For example, in the lightlike case, we set

$$k_+=-i\beta_k\ ,\quad k_-=0\ ,\quad k_i=k\delta_{ia} \quad (3.10)$$

for a quadratic term in a specific direction X^a and then sum over the contributions for N such directions. In the following, we use subscripts i to represent all (transverse) spatial dimensions, while subscripts a represent only those spatial dimensions on which the tachyon depends quadratically.

In the lightlike tachyon background (2.4,2.5), the shift in the dilaton equation of motion becomes

$$\begin{aligned}\frac{\delta(\Delta S)}{\delta\phi} =\ & 2\gamma^2T(X)e^{-2V\cdot X}\Big\{4D\phi+\alpha'\left[(\partial_++\beta)(\partial-2V)_-\phi+\partial_-(\partial-2V)_+\phi-\partial_i\partial_i\phi\right]\\ &+\frac{1}{4}\alpha'^2\beta^2\partial_-(\partial-2V)_-\phi\Big\}+\frac{1}{2}\gamma^2\alpha'\mu^2e^{\beta X^+}e^{-2V\cdot X}\Big\{-\partial_a\partial_a\phi+\frac{1}{2}N\beta(\partial-2V)_-\phi\\ &-\frac{1}{2}N\partial_-\partial_+\phi+\frac{1}{4}N\partial_i\partial_i\phi+\frac{1}{4}(\partial-2V)^2\phi\Big\}\ -2\gamma^2\mu^2e^{\beta X^+}e^{-2V\cdot X}X^a\Big\{\partial_a\phi\\ &-\frac{1}{4}\alpha'\beta(\partial-2V)_-\partial_a\phi-\frac{1}{4}\alpha'\beta\partial_-\partial_a\phi\Big\}\ . \end{aligned}\quad (3.11)$$

The timelike tachyon background (2.9) yields a similar result, which is not illustrative enough to repeat. Now we note that the complete dilaton equation of motion in the linear dilaton background is

$$\frac{\delta S}{\delta\Phi}=\sqrt{-g}e^{-2\Phi}\left[-8V^2-2R+8\partial_\mu\Phi\partial^\mu\Phi-8\nabla^2\Phi\right]\ , \quad (3.12)$$

where $4V^2$ is the cosmological constant due to the supercritical dimension. We write $\Phi=\Phi_0+V_\mu X^\mu+\kappa\phi$ and linearize to find the equation of motion for fluctuations at zeroth order in the tachyon background.[4] Combining, we find the equation of motion to first order in the tachyon background, which is

$$\begin{aligned}0 =\ & \left(\partial^2-2V\cdot\partial\right)\phi-\frac{1}{4}\gamma^2T(x)\left\{4D\phi+\alpha'\beta(\partial-2V)_-\phi+\frac{1}{4}\alpha'^2\beta^2\partial_-(\partial-2V)_-\phi\right\}\\ &-\frac{1}{16}\gamma^2\mu^2\alpha' e^{\beta X^+}\left\{\frac{1}{2}N\beta(\partial-2V)_-\phi-\partial_a\partial_a\phi+\frac{1}{2}NV\cdot\partial\phi-\frac{1}{2}V\cdot(\partial-2V)\phi\right\}\\ &+\frac{1}{4}\gamma^2\mu^2e^{\beta X^+}X^a\partial_a\left\{\phi-\frac{1}{4}\alpha'\beta(\partial-2V)_-\phi-\frac{1}{4}\alpha'\beta\partial_-\phi\right\}\end{aligned}\quad (3.13)$$

[4] κ, the D-dimensional Planck constant, is included so that the fluctuation ϕ has canonical dimension (and normalization up to constants of order unity).

for the lightlike tachyon. Note that, since we are working to first order in the tachyon, we were able to remove terms from (3.11) that are proportional to the linearized (3.12).

For completeness, we also list the linearized dilaton equation of motion for the timelike tachyon background. It is

$$\begin{aligned}0 =\ & \left(\partial^2 - 2V\cdot\partial\right)\phi - \frac{1}{4}\gamma^2 T(X)\left\{4D\phi + \alpha'\beta(\partial-2V)_0\phi + \frac{3}{8}\alpha'^2\beta^3(\partial-2V)_0\phi\right.\\ & \left.+\frac{3}{16}\alpha'^2\beta^4\phi + \frac{1}{2}\alpha'^2\beta^3 V_0\phi + \frac{1}{4}\alpha'^2\beta^2 V_0^2\phi + \frac{1}{4}\alpha'^2\beta^2\partial_0(\partial-2V)_0\phi\right\}\\ & -\frac{1}{16}\gamma^2\mu^2\alpha' e^{\beta X^0}\left\{\frac{1}{2}N\beta(\partial-2V)_0\phi - NV_0^2\phi - \frac{3}{2}N\beta^2\phi - \partial_a\partial_a\phi - 2N\beta V_0\phi\right\}\\ & +\frac{1}{4}\gamma^2\mu^2 e^{\beta X^0}X^a\partial_a\left\{\phi - \frac{1}{8}\alpha'\beta^2\phi - \frac{1}{4}\alpha'\beta(\partial-2V)_0\phi - \frac{1}{4}\alpha'\beta\partial_0\phi\right\}\ .\end{aligned} \tag{3.14}$$

Before moving on, we should pause to reflect on the various parts of (3.13,3.14). First of all, there are time dependent mass terms, including contributions from the fact that the vertex operator for ϕ includes polarizations in the N spatial directions annihilated by the tachyon. Next, there are time dependent drag terms due to the tachyon gradient and an extra contribution due to momentum in the X^a directions. Also, there are terms with second order (lightcone) time derivatives, which are reminiscent of the shift in the spacetime metric found in [25,29] through renormalization of the worldsheet theory. Finally, there are terms of the form $X^a P_a$. Many of these terms were anticipated by the Hamiltonian calculation carried out in [41].

4. Backreaction

We now have the pieces we need to study quantum backreaction effects in tachyon condensation, working self-consistently in the region of spacetime that has both small string coupling and small tachyon condensate. We will see that the backreaction due to quantum particle production can become strong even in that region.

As a brief review, [41] studied particle creation in the time-reversed background using an approximate equation of motion derived with a simplified Hamiltonian treatment of the string, concluding that the tachyon "decondensation" resulted in a thermal bath at inverse temperature β, which is about the string scale, created as the tachyon vanishes. Since the Bogoliubov coefficients are simply conjugated under time reversal, we expect a thermal state of temperature $1/\beta$, but this thermal bath is created as the tachyon becomes strong. We actually find a much stronger source of backreaction. We will begin by solving the dilaton fluctuation equation (3.13,3.14) in a truncated form; then we present numerical calculations of the strength of the backreaction.

4.1 Solutions

We are most interested in the time dependence of the dilaton equation of motion (3.13,3.14), so we truncate the somewhat complicated spatial dependence (that is, the appearance of both

spatial derivatives and spatial positions in the differential equation). Specifically, we will work with

$$\left[\partial_t^2 - 2V_t\partial_t + k^2 + m^2 e^{\beta t}\left(c_1 + c_2\partial_t\right)\right]\phi = 0\ , \tag{4.1}$$

where $t = X^0$ is the usual time coordinate. We obtain this simplified equation of motion by setting X^a to a small value in either (3.13) or (3.14), then treating X^a as a constant while Fourier transforming to momentum space. (In addition, we set $X^1 = 0$ and dropped X^1 derivatives in the lightlike gradient case.) We also ignore the subleading (nonexponential) time dependence in the tachyon profile T. We have also combined many of the constants into single variables (m^2, c_1, c_2) for notational convenience. It is also helpful, of course, to rewrite the equation of motion in terms of $x = \beta t$, which gives

$$\left[\partial_x^2 + 2V\partial_x + k^2/\beta^2 + M^2 e^{\beta t}\left(a + b\partial_x\right)\right]\phi = 0\ . \tag{4.2}$$

For reference, the constants appearing in (4.2) are

$$\begin{aligned} V &= |V_t/\beta| = \alpha' V_t^2/2 = (D-26)/12 \\ M^2 &= \gamma^2\mu^2(X^aX^a)/8\alpha'\beta^2 \end{aligned} \tag{4.3}$$

In the lightlike case,

$$\begin{aligned} a &= 4D - 2\alpha'\beta^2 V_t - \frac{1}{4}\alpha'^2\beta^2k^2 - \left(\frac{\alpha'^2}{(X^a)^2}\right)\left(\frac{1}{2}V_t^2 + \frac{1}{2}N\beta V_t\right) \\ b &= \alpha'\beta^2 + \left(\frac{\alpha'^2}{(X^a)^2}\right)\left(\frac{1}{4}N\beta^2 - \frac{1}{4}(2N+1)\beta V_t\right)\ , \end{aligned} \tag{4.4}$$

and, in the timelike case,

$$\begin{aligned} a &= 4D - 2\alpha'\beta^2 V_t - \frac{1}{4}\alpha'^2\beta^2k^2 - \frac{1}{4}\alpha'^2\beta^3V_t + \frac{3}{16}\alpha'^2\beta^4 + \frac{1}{4}\alpha'^2\beta^2V_t^2 \\ &\quad - \left(\frac{\alpha'^2}{(X^a)^2}\right)\left(\frac{3}{2}N\beta V_t + \frac{1}{2}NV_t^2 + \frac{3}{4}N\beta^2\right) \\ b &= \alpha'\beta^2 + \frac{3}{8}\alpha'^2\beta^3 + \left(\frac{\alpha'^2}{(X^a)^2}\right)\left(\frac{1}{4}N\beta^2\right)\ . \end{aligned} \tag{4.5}$$

In our study, we will study a simplifying limit in which the parameters a and b coincide for the lightlike and timelike tachyon cases. First, we work in the large D limit (which is where the timelike tachyon background is reliable anyway). We also set $N \sim 1$ and $(X^a)^2 \gg D\alpha'$. In this limit,

$$a = 4D\ ,\quad b = \alpha'\beta^2 \tag{4.6}$$

in either case; the most important parameters for us will turn out to be $V = D/12$ and $a/b = D^2/6 = 24V^2$ in this limit. It is also important to note two features. First is that M^2 may take either sign, depending on the sign of the tachyon (recall that μ^2 may be positive or negative). Second, note that the tachyon amplitude is controlled by $M^2 b$.

The solution to (4.2) is

$$\phi\left(x,\vec{k}\right) = e^{-Vx}\left[Ae^{\nu x}M\left(\nu+\frac{a}{b};1+2\nu;M^2be^x\right)+Be^{-\nu x}M\left(-\nu+\frac{a}{b};1-2\nu;M^2be^x\right)\right] \ , \tag{4.7}$$

where $\nu = \sqrt{V^2-k^2/\beta^2}$. The function M, also denoted ${}_1F_1$, is the Kummer or confluent hypergeometric function. These solutions match to positive and negative frequency modes in the far past, with frequency given by ν, though those modes are real exponentials at long wavelengths (real ν). Both the solutions grow rapidly as $\exp[M^2be^x]$ in the future, but the tachyon is no longer a perturbation of the background by then; we can trust (4.7) only for $t < -(1/\beta)\ln M^2b$.

4.2 Source Term Estimates

To study the backreaction of quantum fluctuations of the dilaton, we can examine the dilaton equation of motion in the linear dilaton background. Including the expectation of quantum fluctuations, this is

$$V_\mu V^\mu + \nabla^2\Phi - \partial_\mu\Phi\partial^\mu\Phi - \langle\kappa^2\partial_\mu\phi\partial^\mu\phi\rangle = 0 \ , \tag{4.8}$$

where Φ is the background dilaton plus fluctuation $\Phi = \Phi_0 + V_\mu X^\mu + \kappa\phi$. ($\kappa$ is given by the D-dimensional Planck scale since ϕ is of canonical dimension while Φ is dimensionless.) The quantum fluctuation yields a source term through the last term in (4.8); backreaction will be important when $\langle\kappa^2(\partial\phi)^2\rangle \gtrsim D/\alpha' \sim |V_\mu V^\mu|$

The expectation value $\langle\kappa^2(\partial\phi)^2\rangle$ is given by the average of the fluctuations on the length scale $1/V_t$, the only scale of the cosmological linear dilaton background. In particular, we choose the momentum scale V_t rather than β because V_t, like the Hubble scale in cosmological backgrounds, is the scale at which fluctuations switch from frozen or growing behavior to oscillatory behavior. (In fact, short wavelength fluctuations behave just like free massless fields, which do not contribute to the source term at all.) The reader may wonder if we can trust our equation of motion (4.2) at wavenumbers beyond $1/\sqrt{\alpha'}$ when $|V_t| > 1/\sqrt{\alpha'}$ since α' corrections should enter at that wavenumber. We will address this point below.

We see that we need to calculate

$$\langle\kappa^2(\partial\phi)^2\rangle = \kappa^2\int_0^{|V_t|}\frac{dk}{(2\pi)^{D-1}}k^{D-2}\left(k^2|\phi|^2-|\dot{\phi}|^2\right) \ . \tag{4.9}$$

The momentum modes in this integral are precisely those with the real exponential behavior in the far past; in order to avoid large backreaction at infinite wavelength in the far past, we require that the integration constant $B=0$. Then, for proper normalization, we should take $A=\sqrt{-i/\beta\nu}$. (If we want to start in a vacuum state at the point where the string coupling of the linear dilaton becomes small, we should have a small admixture of B, but this should not change our results significantly.)

Defining

$$\phi(x) = \sqrt{-\frac{i}{\beta\nu}}e^{-Vx}\sigma(z) \ , \quad z = M^2be^x \ , \tag{4.10}$$

we find

$$\langle \kappa^2 (\partial\phi)^2 \rangle = \frac{\kappa^2 \beta^D e^{-2Vx}}{(2\pi)^{D-1}} \int_0^V d\nu \left(V^2 - \nu^2\right)^{(D-3)/2} \left(\left(V^2 - \nu^2\right) |\sigma|^2 - |V\sigma - z\partial_z \sigma|^2 \right) . \quad (4.11)$$

To determine the extent of the backreaction when the tachyon becomes important, we should study this integral at $z = \pm 1$. This is easiest to approximate for a large dimensionality D. As discussed at the end of the last subsection, $V = D/12$ and $a/b = 24V^2$ in that limit. Finally, the Planck scale is given by $\kappa^2 \sim (2\pi)^{D-3} \alpha'^{(D-2)/2} e^{2\Phi_0}$, so we find

$$\langle \kappa^2 (\partial\phi)^2 \rangle \approx \frac{g_s^2}{\alpha'} \left(\frac{24}{D}\right)^{D/2} I_D \ , \quad (4.12)$$

with I_D the integral from (4.11). Here, g_s is the string coupling at the time we study, when the tachyon amplitude $M^2 b e^x$ reaches order unity.

We have integrated I_D numerically for several values of the dimensionality up to $D = 480$ for both signs of the tachyon (*i.e.*, $z = \pm 1$). We find that I_D grows much more quickly than $D^{D/2}$ in either case; in fact, even if we ignore the factors of 2π in the Planck scale, the backreaction can still become large, since I_D appears to grow even faster than $(2\pi)^D D^{D/2}$. For example, I_D becomes larger than $(2\pi)^D D^{D/2}$ for $z = -1$ at $D \gtrsim 240$, while it is 167 orders of magnitude larger for $z = 1$ at that dimensionality. These results are summarized in table 1. We stress that the top row gives the best estimate of the source term; the second row is given as a (very) conservative lower limit.

Dimension		48	72	96	120	144	168	192	216	240	360	480
$\log(I_D(24/D)^{D/2})$	$z=1$	53	87	123	160	198	237	277	317	358	571	792
	$z=-1$	19	40	54	74	95	118	140	163	187	315	452
$\log(I_D(24/4\pi^2 D)^{D/2})$	$z=1$	15	30	46	64	83	103	124	145	167	284	409
	$z=-1$	-20	-17	-22	-22	-20	-16	-13	-9	-4	28	69

Table 1: The (base-10 log of the) integral I_D calculated with $V = D/12$ and $a/b = D^2/6$ for a range of dimensionalities.

As mentioned above, a cautious reader will not trust equation (4.2) for wavenumbers $k \gtrsim 1/\sqrt{\alpha'}$ because string worldsheet corrections should become important at that scale. Therefore, to be cautious, we consider summing over only momenta up to $k = 1/\sqrt{\alpha'}$. There are two points to make before presenting the results. First, V_t is smaller than the string scale as long as $D \leq 182$, so the correct results are those of (1) for those dimensions. Second, it may not be necessary to cut off the integral at $k \sim 1/\sqrt{\alpha'}$ if we are only concerned with time dependence, since the time derivative of the dilaton fluctuation is always smaller than the linear dilaton gradient, the natural scale of the problem. In any case, though, to be cautious, we present results at some large dimensions with a upper momentum cut-off of $k = 1/\sqrt{\alpha'}$. This just changes the upper integration limit of (4.9) to $1/\sqrt{\alpha'}$. In equation (4.11), the lower integration limit becomes $\nu = \sqrt{V^2 - 1/\alpha'\beta^2} = \sqrt{V^2 - V/2} \approx V - 1/4$. Adding this lower

limit restricts the backreaction a great deal. In the case of negative tachyon sign ($z = -1$), we find that the backreaction is neglible for large D (and decreasing as D increases). However, the negative sign tachyon still gives a large (and increasing) source term at large D. These results are summarized in table 2.

Dimension		192	216	240	360	480	540	600	660
$\log(I_D(24/D)^{D/2})$	$z=1$	80	90	100	151	201	226	251	277
	$z=-1$	-55	-60	-67	-100	-133	-149	-166	-183

Table 2: The integral I_D calculated with $V = D/12$ and $a/b = D^2/6$ for a range of dimensionalities. In this case, the lower limit of the integral is $V - 1/4$.

4.3 Discussion

From the above calculations, it appears that the only way to avoid (severe) backreaction effects at large D is to tune the string coupling to be extremely small at the time the tachyon becomes strong. We stress that backreaction can be quite significant well into the region of perturbative string coupling. The backreaction is, in fact, perturbative in the string coupling, but the coefficient of g_s^2 is much larger than unity.

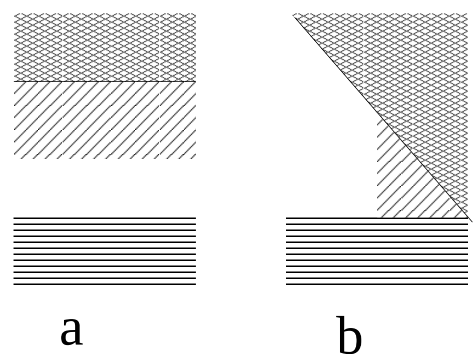

Figure 2: Regions of strong coupling (horizontal shading), strong tachyon (cross-hatched shading), and possible strong backreaction (diagonal shading) for timelike (a) and lightlike (b) tachyon gradients.

In the case of timelike tachyon gradient, this is not burdensome, since we usually assume the strong coupling regime to be far in the past in order to use weakly coupled string theory. Keeping the backreaction under control just means that we push the strongly coupled region even further into the far past. As a caution, though, just due to the large numbers involved, it seems like backreaction may be important even at times well before the tachyon background becomes strong. In this case, we can ask whether any effects can propagate from a region of significant backreaction. Since any string matter generated through backreaction only generates effects of order g_s or higher, we expect that having the strong tachyon region in sufficiently weak string coupling will render backreaction effects negligible. However, it is possible to imagine a scenario in which the backreaction (in a region of both weak coupling and weak tachyon) could either cause the dilaton to grow again or accelerate the growth of the tachyon, leading to difficulties with the usual picture. Of course, it seems that ensuring the string coupling is sufficiently weak for a very long time before the tachyon becomes important can control backreaction effects. Therefore, it is important to stress that the strong coupling era must end quite some time before the tachyon becomes strong. As a reference, in table 3, we list the number of string times the strong coupling and strong tachyon

regions must be separated by (at a bare minimum) to avoid strong backreaction effects in some cases.

The situation is slightly more complicated in the case of a lightlike tachyon gradient. In that case, there is always a region where the tachyon is large and the dilaton is not extremely small. In other regions where the tachyon is strong, of course, the dilaton *will* be extremely small, so backreaction will be negligible; the main question is whether any effects can propagate from the region of important backreaction, as discussed above. Once again, we expect that sufficiently small string coupling will control the backreaction. The regions of significant backreaction are summarized in figure 2.

Dimension		48	72	96	120	144	168	192	216	240	360	480
Δt	$z = 1$	15	17	18	18	19	19	20	20	21	22	23
	$z = -1$	5	8	8	8	9	10	10	10	11	12	13

Table 3: The number of string times needed to separate the strong coupling and strong tachyon regions in order to avoid backreaction at the time of strong tachyon. These results use the calculations in table 1.

Of course, we have so far only carried out a rough calculation of the backreaction in the bosonic string. However, the strength of our results indicates that backreaction is a serious issue that may complicate the picture of closed string tachyon condensation, and these concerns will certainly also arise in the heterotic case. Sadly, backreaction may spoil the clean picture of the closed string tachyon as smoothly annihilating dimensions of spacetime, at least in some regions. To gain a more complete understanding of closed string tachyon condensation, we will need to understand backreaction due to the production of massless string modes in the tachyon background. In the meanwhile, our results serve as a caveat for interpreting tachyon condensation.

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A. BRST Quantization in Linear Dilaton Background

For reference, we provide here the BRST quantization of a closed string at the tachyonic and massless levels in a linear dilaton background. The results are necessary for us to simplify the string amplitude calculated in section 3.1. We largely follow the discussion for the Minkowski string in [44].

On the complex plane, the holomorphic and antiholomorphic worldsheet stress tensors are just given by the (Euclideanization of) 2.15 in the linear dilaton background. Therefore, the first few Virasoro generators in the scalar sector are

$$\begin{aligned} L_0^X &= \frac{1}{2\pi\alpha'}\oint dz\, zT(z) = \frac{\alpha'}{4}p^2 + \sum_{n=1}^{\infty}\alpha_{-n}\alpha_n + i\frac{\alpha'}{2}V_\mu p^\mu \\ L_1^X &= \frac{1}{2\pi\alpha'}\oint dz\, z^2T(z) = \frac{1}{2}\sum_{n=-\infty}^{\infty}\alpha_{1-n}\alpha_n + i\sqrt{2\alpha'}V_\mu\alpha_1^\mu \\ L_{-1}^X &= \frac{1}{2\pi\alpha'}\oint dz\, T(z) = \frac{1}{2}\sum_{-\infty}^{\infty}\alpha_{-1-n}\alpha_n\ . \end{aligned} \tag{A.1}$$

The antiholomorphic generators are just the complex conjugates, as usual. We also need the contribution to the Virasoro operators from the ghosts, which we can copy from [44]. These are

$$L_0^g = \sum_{n=-\infty}^{\infty} n\!:\!b_{-n}c_n\!:,\ L_1^g = \sum_{n=-\infty}^{\infty}(2-n)\!:\!b_nc_{1-n}\!:,\ L_{-1}^g = -\sum_{n=-\infty}^{\infty}(2+n)\!:\!b_nc_{-1-n}\!:. \tag{A.2}$$

Here the normal ordering symbol stands for creation-annihilation normal ordering, unlike in the main text.

The BRST operator for the linear dilaton is then

$$Q_B = \sum_n\left(c_nL_{-n}^X + \tilde{c}_n\tilde{L}_{-n}^X\right) + \sum_{m,n}\frac{m-n}{2}\left(:c_mc_nb_{-m-n}: + :\tilde{c}_m\tilde{c}_n\tilde{b}_{-m-n}:\right) - c_0 - \tilde{c}_0\ . \tag{A.3}$$

A physical state $|\psi\rangle$ of the string must be in the cohomology of Q_B and satisfy $b_0|\psi\rangle = \tilde{b}_0|\psi\rangle = 0$ and $L_0|\psi\rangle = \tilde{L}_0|\psi\rangle = 0$. We can ensure the b_0 condition just by taking the appropriate ghost ground state; since $L_0 = \{Q_B, b_0\}$, the L_0 conditions follow. In practice, however, we'll find it instructive to examine the complete L_0 condition, which works out to be

$$L_0|\psi\rangle = 0 \Rightarrow \quad p^2 + 2iV\cdot p = -\frac{4}{\alpha'}(N-1)\ , \tag{A.4}$$

where N is the total holomorphic matter plus ghost oscillator excitation number. The antiholomorphic sector gives the same condition with $N \to \tilde{N}$, but level matching requires $N = \tilde{N}$.

At the tachyonic level, the state of the string can only be $|0;k\rangle$ with $k^2 + 2iV\cdot k = 4/\alpha'$. In addition,

$$Q_B|0;k\rangle = \left(c_0L_0 + \tilde{c}_0\tilde{L}_0\right)|0;k\rangle = 0 \tag{A.5}$$

just because of the mass-shell condition.

At the massless level ($k^2 + 2iV\cdot k = 0$), the most general state is

$$\begin{aligned} |\psi\rangle = &\Big(e_{\mu\nu}\alpha_{-1}^\mu\tilde{\alpha}_{-1}^\nu + f_\mu\alpha_{-1}^\mu\tilde{b}_{-1} + \tilde{f}_\mu b_{-1}\tilde{\alpha}_{-1}^\mu + g_\mu\alpha_{-1}^\mu\tilde{c}_{-1} + \tilde{g}_\mu c_{-1}\tilde{\alpha}_{-1}^\mu \\ &+hb_{-1}\tilde{c}_{-1} + \tilde{h}c_{-1}\tilde{b}_{-1} + \beta b_{-1}\tilde{b}_{-1} + \gamma c_{-1}\tilde{c}_{-1}\Big)|0;k\rangle\ . \end{aligned} \tag{A.6}$$

The BRST operator on this state is

$$Q_B|\psi\rangle = \sqrt{\frac{\alpha'}{2}}\Big[e_{\mu\nu}(k+2iV)^\mu c_{-1}\tilde{\alpha}^\nu_{-1} + e_{\mu\nu}(k+2iV)^\nu \alpha^\mu_{-1}\tilde{c}_{-1} + f\cdot(k+2iV)c_{-1}\tilde{b}_{-1} + f\cdot\alpha_{-1}k\cdot\tilde{\alpha}_{-1}$$
$$+\tilde{f}\cdot(k+2iV)b_{-1}\tilde{c}_{-1} + \tilde{f}\cdot\tilde{\alpha}^\mu_{-1}k\cdot\alpha_{-1} + g\cdot(k+2iV)c_{-1}\tilde{c}_{-1} + \tilde{g}\cdot(k+2iV)c_{-1}\tilde{c}_{-1}$$
$$+hk\cdot\alpha_{-1}\tilde{c}_{-1} + \tilde{h}c_{-1}k\cdot\tilde{\alpha}_{-1} + \beta k\cdot\alpha_{-1}\tilde{b}_{-1} + \beta b_{-1}k\cdot\tilde{\alpha}_{-1}\Big]|0;k\rangle\ . \tag{A.7}$$

If $|\psi\rangle$ is BRST-closed, then we must have

$$e_{\mu\nu}(k+2iV)^\mu + \tilde{h}k_\nu = 0\ ,\quad e_{\mu\nu}(k+2iV)^\nu + hk_\mu = 0\ ,\ \beta = 0\ ,$$
$$f_\mu = \tilde{f}_\mu = 0\ ,\ g\cdot(k+2iV) = \tilde{g}\cdot(k+2iV)\ . \tag{A.8}$$

The general BRST-exact state at this level is of the form (A.7) with primed coefficients. By choosing f'_μ and $\tilde{f}'_\mu$, we can therefore gauge away h and $\tilde{h}$ (compare terms in (A.6) and (A.7)). By choosing $e'_{\mu\nu}$, we can gauge away g_μ and $\tilde{g}_\mu$. Finally, by choosing g'_μ and $\tilde{g}'_\mu$, we can gauge away γ. We are required to choose $\beta' = 0$ to maintain the condition $f_\mu = \tilde{f}_\mu = 0$. Once we have made these choices, we can make a further transformation with f''_μ and $\tilde{f}''_\mu$ as long as $f''\cdot(k+2iV) = \tilde{f}''\cdot(k+2iV) = 0$, which shifts $e_\mu\nu$. We are left with the following state, conditions, and gauge equivalence:

$$|\psi\rangle = e_{\mu\nu}\alpha^\mu_{-1}\tilde{\alpha}^\nu_{-1}|0;k\rangle$$
$$0 = e_{\mu\nu}(k+2iV)^\mu = e_{\mu\nu}(k+2iV)^\nu$$
$$e_{\mu\nu} \simeq e_{\mu\nu} + \sqrt{\frac{\alpha'}{2}}f''_\mu k_\nu + \sqrt{\frac{\alpha'}{2}}k_\mu\tilde{f}''_\nu\ . \tag{A.9}$$

The polarization tensor $e_{\mu\nu}$ can then be separated into a symmetric graviton, 2-form potential, and dilaton trace parts, as normal.

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